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Examination

Electrodynamics, TFYY67
2009-03-11
Elektromagnetisk fältteori och vågutbredning

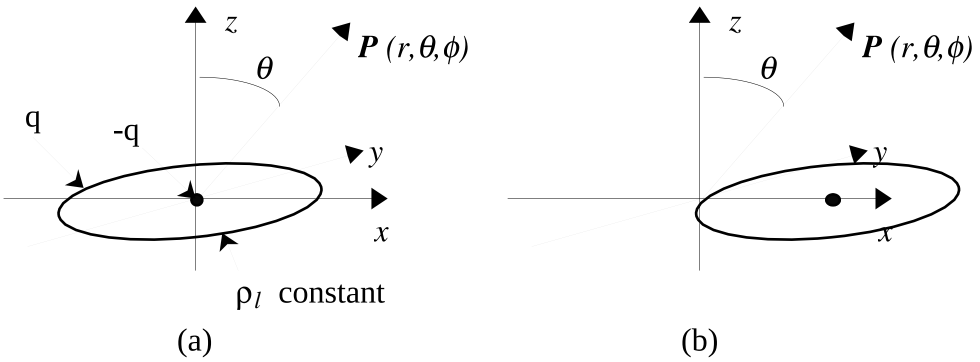
The examination consists of 4 problems. Each correctly solved problem gives 4 points. The points you have earned from solving the home-work problems will be added to the results of the examination. The grades will be set according to:

grade 3: total score of 8-11 points
grade 4: 12-15 points
grade 5: 16-20 points

Allowed to bring to the examination: "Classical Electromagnetic Radiation" by Heald & Marion, Physics Handbook, English dictionary, electronic calculator.
Additional material might be distributed during the examination.

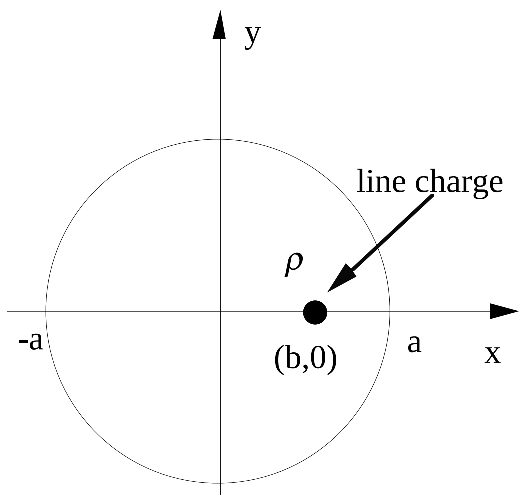
The solutions should be in either English or Swedish.

1. (4p) a) Make a multipole expansion of the potential from the charge distribution in figure (a). The total charge q is evenly distributed on a ring of radius a, centered around the origin. The point charge -q is placed at the origin. Include contributions up to and including the quadrupole term.



b) Shift the ring and point charge the distance a in the x-direction (see fig (b)). Will the result change? Motivate in words or demonstrate with calculations.

2. (4p) Use conformal mapping to find the potential inside an electrically conducting tube of radius a kept at zero potential. A line charge, of charge density ρ, is placed at distance $b = sa$, $0 < s < 1$, from the center of the tube.



Hint: Use a möbius transformation that maps the circle into a unit circle, with center at the origin, and that maps the line charge onto the origin. Solve for the potential in the new geometry and then use the inverse transformation to get the potential of the original problem.

3. (4p) Place a grounded conducting sphere of radius r_0 centered around the origin. The grounding can be thought of as being achieved by connecting the sphere with a thin electrically insulated wire with a big object far away having zero potential (the ground of the earth for example). Neglect all effects from the wire except that it makes the sphere grounded. Now introduce a charge q at a distance R from the origin, R being larger than r_0. A current will result in the wire and the sphere will be charged. While the charge is present cut and remove the wire. Then remove the charge q.

a) calculate how much charge has accumulated on the sphere.

b) calculate the potential of the sphere.

Hint: The method of mirror images should be suitable to use here.

4. (4p) One often introduces potentials in the solution of electromagnetic problems. The **E** and **B** fields are then found from these using the relations:

$$\mathbf{E} = -\nabla\Phi - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}$$
$$\mathbf{B} = \nabla \times \mathbf{A}$$

We have shown that the scalar and vector potentials in Coulomb gauge (derived from the assumption $\nabla \cdot \mathbf{A} = 0$) are solutions to the following differential equations:

$$\nabla^2\Phi = -4\pi\rho$$
$$\nabla^2\mathbf{A} - \frac{1}{c^2}\frac{\partial^2 \mathbf{A}}{\partial t^2} = -\frac{4\pi}{c}\mathbf{J}_\perp$$

Show that in this gauge:

a) the contribution to the **E** field from the scalar potential is purely longitudinal.
b) the contribution to the **E** field from the vector potential is purely transverse.
c) the **B** field is purely transverse.