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Examination

**Electrodynamics, TFYY67
2008-06-03**

Elektromagnetisk fältteori och vågutbredning

The examination consists of 4 problems. Each correctly solved problem gives 4 points. The points you have earned from solving the home-work problems will be added to the results of the examination. The grades will be set according to:

grade 3: total score of 8-11 points

grade 4: 12-15 points

grade 5: 16-20 points

Allowed to bring to the examination: “Classical Electromagnetic Radiation” by Heald & Marion, Physics Handbook, English dictionary, electronic calculator.

Additional material might be distributed during the examination.

The solutions should be in either English or Swedish.

1. The resulting potential from a point charge, of charge q_0 , placed at the origin in a conducting medium is in the Thomas-Fermi approximation given by the expression:

$$\Phi(r) = q_0 \frac{e^{-r/a}}{r}$$

where the constant a depends on the material parameters (the density, charge and mass of the carriers responsible for the screening).

Find

a) the induced charge density, $\rho_{ind}(r)$, from the carriers responsible for the screening. The charge density meant is charge per unit volume.

b) the total induced charge:

$$q_{ind} = \int d^3r \rho_{ind}(\mathbf{r})$$

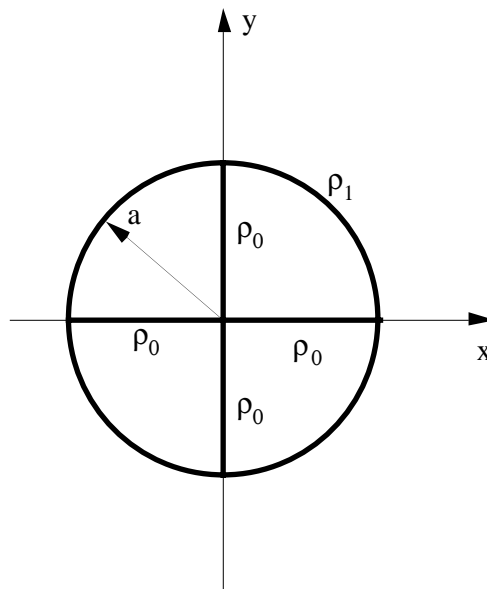
Hint: Note that the resulting potential above has contributions both from the original point charge and from the screening charge.

2. Calculate

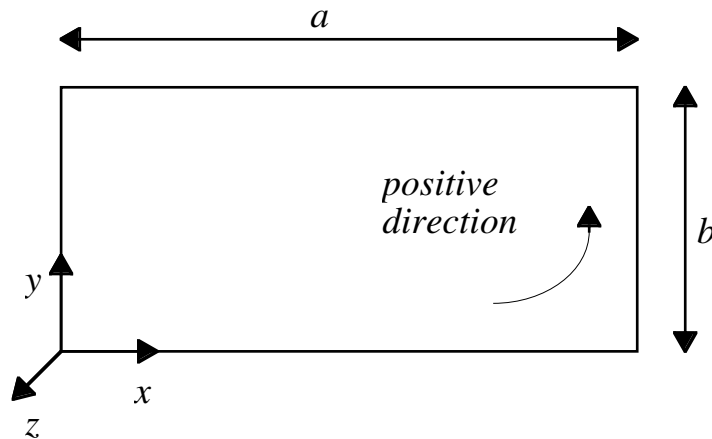
a) the monopole moment, dipole moment, and quadrupole tensor for the system in the figure. The charges are confined to the xy -plane and the line charge densities are:

$$\rho_0 = q/a \quad ; \quad \rho_1 = -2q/\pi a$$

b) the multipole expansion of the potential, expressed in polar coordinates, up to and including the quadrupole contributions.



3. Let the electric field be $\mathbf{E} = E_0 [(\rho/\rho_0) \cos \varphi \hat{\rho} + \sin \varphi \hat{\varphi}]$.
- a) Evaluate $\oint_{\Gamma} \mathbf{E} \cdot d\mathbf{l}$ around the path, Γ , enclosing the surface, S , defined by $z = 0$; $2\rho_0 \leq \rho \leq 5\rho_0$; $30^\circ \leq \varphi \leq 60^\circ$. The surface normal is \hat{z} .
- b) Confirm this using Stoke's theorem.
4. The fields $\mathbf{E} = A \sin(kx - \omega t) \hat{y}$ and $\mathbf{B} = A \sin(kx - \omega t) \hat{z}$ form together a solution to Maxwell's equations in vacuum. We are looking for the electromotive force (EMF) that is induced in a metal wire with position and shape according to the figure. This EMF can be calculated in two different ways according to one of Maxwell's equations. The objective is to show this by explicit calculations.



- a) Calculate the line integral $\text{EMF} = \oint_C \mathbf{E} \cdot d\mathbf{l}$, where C is the curve around the rectangle.
- b) Calculate the line integral $\text{EMF} = \frac{1}{c} \int_S -\frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S} = -\frac{1}{c} \frac{\partial \Phi}{\partial t}$, where S is the surface of the rectangle in the plane of the paper.
- c) This is a simple receiving antenna. Different values of a give different amplitude for the EMF. For which values of a do we get best signal? For which values of a does the signal disappear completely? Express the answer in λ .