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# Examination

**Electrodynamics, TFYY67  
2008-03-11**

**Elektromagnetisk fältteori och vågutbredning**

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The examination consists of 4 problems. Each correctly solved problem gives 4 points. The points you have earned from solving the home-work problems will be added to the results of the examination. The grades will be set according to:

**grade 3: total score of 8-11 points**

**grade 4: 12-15 points**

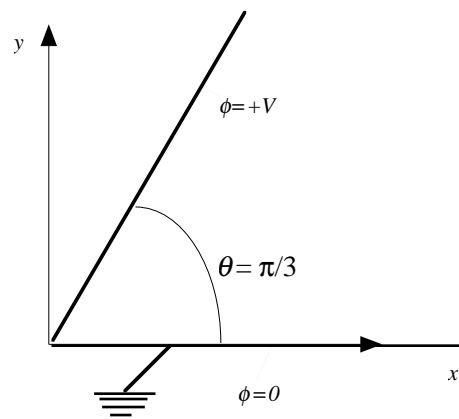
**grade 5: 16-20 points**

**Allowed to bring to the examination: “Classical Electromagnetic Radiation” by Heald & Marion, Physics Handbook, English dictionary, electronic calculator.**

**Additional material might be distributed during the examination.**

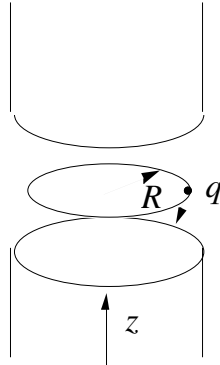
**The solutions should be in either English or Swedish.**

1. (4p) An electric dipole is aligned along the  $z$ -axis. It consists of a charge  $+q$  at  $z = a$  and a charge  $-q$  at  $z = -a$ . Make a multipole expansion of the resulting potential up to and including the octupole contribution. Give the results in spherical coordinates.
  
2. (4p) Two metallic half planes are positioned according to the figure. There is a small gap between them along the  $z$ -axis.
  - a) (2p) Determine the potential between the metallic planes.
  - b) (2p) Determine the induced charge density on the top of the horizontal plane, i.e., the one in the  $xz$ -plane.



3. Let the electric field be  $\mathbf{E} = E_0 \left[ \left( \rho / \rho_0 \right) \cos \varphi \hat{\rho} + \sin \varphi \hat{\varphi} \right]$ .
  - a) (2p) Evaluate  $\oint_S \mathbf{E} \cdot d\mathbf{S}$  over the surface,  $S$ , enclosing the volume,  $V$ , defined by  $0 \leq z \leq z_0$ ;  $2\rho_0 \leq \rho \leq 5\rho_0$ ;  $30^\circ \leq \varphi \leq 60^\circ$ .
  - b) (2p) Confirm this using Gauss' (divergence) theorem.

4. (4p) A charge  $q$  is moving in a circular orbit, of radius  $R$ , about the center of a cylindrically symmetrical magnet, as shown.



Assume that the orbit of the charge lies in the median plane between the poles of the magnet and hence the only component of magnetic field that it sees is in the  $z$  direction. The field is now allowed to increase with time. Show that the particle will accelerate without change in radius if the increase in the average field for  $r \leq R$  is twice the increase in the field at  $r = R$ . That is

$$\frac{d\bar{B}(R)}{dt} = 2 \frac{dB(R)}{dt},$$

where

$$\bar{B}(R) = \frac{1}{\pi R^2} \int_0^R B(R) 2\pi R dR$$

*Note: This is how the Betatron works. The Betatron is a magnetic induction accelerator for electrons.*