

Linköping Institute of Technology  
Department of Physics, Chemistry and Biology  
Bo E. Sernelius  
Tel. 281724  
Mobile: 073 931 9102

2006-03-17

# Examination

**Electrodynamics, TFYY67, NFYD70**  
**March 17, 14.00-18.00**  
**Elektromagnetisk fältteori och vågutbredning**

---

The examination consists of 4 problems. Each correctly solved problem gives 4 points. The points you have earned from solving the home-work problems will be added to the results of the examination. The grades will be set according to:

**grade 3: total score of 8-11 points**

**grade 4: 12-15 points**

**grade 5: 16-20 points**

**grade G: 8-14 points**

**grade VG: 15-20 points**

**Allowed to bring to the examination: “Classical Electromagnetic Radiation” by Heald & Marion, Physics Handbook, English dictionary, electronic calculator.**

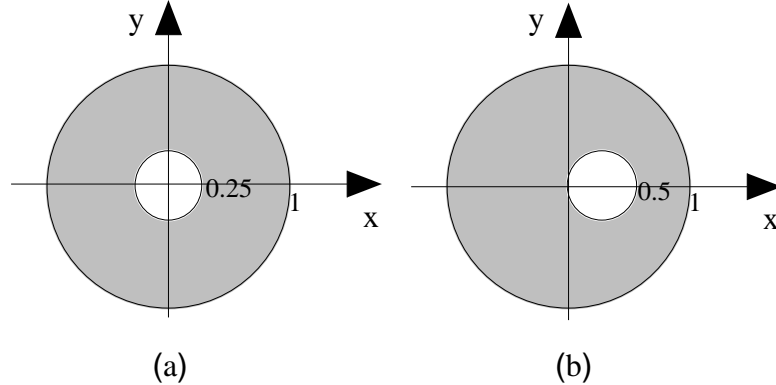
**Additional material might be distributed during the examination.**

**The solutions should be in either English or Swedish.**

- Determine the quadrupole tensor for a sphere of radius  $a$  with center at the origin and the volume charge density given by  $\sigma = bz^4$ .
- A metallic sphere of radius  $a$  is placed with its center at the origin. A line charge is placed along  $(b,y,0)$ ,  $b \geq a$ . Determine the locus of the induced mirror charge (the curve traced out by all the mirror charges).
- Show by explicit derivation (using 14.76 or 14.77 but not 14.68) that  $\mathbf{E} \cdot \mathbf{B}$  is the same in all reference frames. Use

$$\begin{aligned}
 E'_1 &= \gamma(E_1 - \beta B_2) & B'_1 &= \gamma(B_1 + \beta E_2) & \mathbf{E}'_{\perp} &= \gamma \left( \mathbf{E}_{\perp} + \frac{1}{c} \mathbf{v} \times \mathbf{B}_{\perp} \right); & \mathbf{E}'_{\parallel} &= \mathbf{E}_{\parallel} \\
 E'_2 &= \gamma(E_2 + \beta B_1) & B'_2 &= \gamma(B_2 - \beta E_1) & \text{or} & & & \\
 E'_3 &= E_3 & B'_3 &= B_3 & \mathbf{B}'_{\perp} &= \gamma \left( \mathbf{B}_{\perp} - \frac{1}{c} \mathbf{v} \times \mathbf{E}_{\perp} \right); & \mathbf{B}'_{\parallel} &= \mathbf{B}_{\parallel}
 \end{aligned}$$

- The student did his diploma work in a capacitor factory where they are making cylindrical capacitors. They start out with long dielectric cylinders of diameter 2 cm, made from a dielectric with dielectric constant 10. Then they drill holes of various diameter at the center of the cylinders. Finally, they put a gold coating on the two cylinder surfaces. The student was given the task to drill holes of diameter 0.5 cm as illustrated in figure (a). Unfortunately, the student made a mistake and ended up with a shifted hole according to figure (b).



- Determine the capacitance per unit length of the capacitor in figure (a), i.e., the capacitance the student was supposed to produce.
- Determine the capacitance the student ended up with, i.e., the capacitance per unit length of figure (b).

*hint: Use the fact that the capacitance per unit length is unchanged by a conformal transformation of the cross section. Try the conformal transformation:*

*$\omega = (z - a)/(az - 1)$ ,  $a$  a real valued constant. The outer circle is mapped onto the unit circle. Choose the constant in such a way that the inner circle is mapped onto a concentric circle.*