1 Monopole moment: 0 (total charge is zero)

Dipole moment: 0 (inversion symmetry)

$$Q_{ij} = \int_{-\infty}^{+\infty} dx_1 \int_{-\infty}^{+\infty} dx_2 \int_{-\infty}^{+\infty} dx_3 \rho(\mathbf{r})\left(3x_i x_j - r^2\delta_{ij}\right)$$

$$Q_{11} = \int_0^{2\pi} d\varphi a\rho_l \left(3(a\cos\varphi)^2 - a^2\right) = \int_0^{2\pi} d\varphi a^3 \frac{q}{2\pi a}\left(3(\cos\varphi)^2 - 1\right) = a^3 \frac{q}{2\pi a} 2\pi\left(\frac{3}{2} - 1\right) = \frac{1}{2}a^2 q$$

$$Q_{22} = \int_0^{2\pi} d\varphi a\rho_l \left(3(a\sin\varphi)^2 - a^2\right) = \int_0^{2\pi} d\varphi a^3 \frac{q}{2\pi a}\left(3(\sin\varphi)^2 - 1\right) = a^3 \frac{q}{2\pi a} 2\pi\left(\frac{3}{2} - 1\right) = \frac{1}{2}a^2 q$$

$$Q = Q_{33} = \int_0^{2\pi} d\varphi a\rho_l \left(3\cdot 0^2 - a^2\right) = 2\pi a \frac{q}{2\pi a}\left(-a^2\right) = -a^2 q$$

$$Q_{12} = Q_{21} = Q_{13} = Q_{31} = Q_{23} = Q_{32} = 0$$

$$\Phi(r,\theta) = \Phi^{(1)}(r,\theta) + \Phi^{(2)}(r,\theta) + \Phi^{(4)}(r,\theta) + \cdots$$
$$= 0 + 0 + \frac{1}{4}Q\frac{\left(3\cos^2\theta - 1\right)}{r^3} + \cdots = -\frac{q}{4}a^2\frac{\left(3\cos^2\theta - 1\right)}{r^3} + \cdots$$

b) Since the quadrupole contribution is the first non-vanishing contribution in the series it will not change if the origin of the coordinate system is moved. Thus, the answer is that the result will not change.

2 We make a mapping from the z-plane into the ω-plane with a möbius transformation in such a way that the circle with radius a is mapped on the unit circle centered around the origin and the line charge is mapped on the origin. Let $z_1 = sa \to \omega_1 = 0$, $z_2 = a \to \omega_2 = 1$, $z_3 = -a \to \omega_3 = -1$, The following mapping and its inverse do the trick:

$$\omega = -\frac{z - sa}{sz - a} \quad ; \quad z = a\frac{\omega + s}{s\omega + 1} \quad ; \quad z = x + iy \quad ; \quad \omega = u + iy$$

It is the result of the conservation of cross ratio: $\frac{(\omega_1 - \omega_2)(\omega_3 - \omega)}{(\omega_1 - \omega)(\omega_3 - \omega_2)} = \frac{(z_1 - z_2)(z_3 - z)}{(z_1 - z)(z_3 - z_2)}$

The potential in the ω-plane is $\Phi(u,v) = -\rho\ln\left(u^2 + v^2\right)$. The potential of our original problem is $\Phi(x,y) = -\rho\ln\left\{[u(x,y)]^2 + [v(x,y)]^2\right\}$; $u(x,y) = \frac{s\left(x^2 + y^2\right) - xa\left(1 + s^2\right) + sa^2}{s^2\left(x^2 + y^2\right) - 2sax + a^2}$;

$v(x,y) = \frac{ay\left(s^2 - 1\right)}{s^2\left(x^2 + y^2\right) - 2sax + a^2}$ or in a more straightforward and better way:

$$\Phi(x,y) = -\rho\ln\left\{\omega(x,y)\omega^*(x,y)\right\} = -\rho\ln\left\{\frac{(x - sa)^2 + y^2}{(a - sx)^2 + (sy)^2}\right\}$$

3 Following the lecture notes on pages 7:1-7:2 we find with our notation that the image charge q' at the distance R' from the center of the sphere are: $\boxed{q' = -r_0 q/R,\ \ R' = r_0^2/R}$. Thus the answer to a) is that the total accumulated charge on the sphere is $q' = -r_0 q/R$. After the wire has been cut the total charge of the sphere stays constant. When the external charge is removed the charge of the sphere distributes itself homogeneously over the outer surface of the sphere. The resulting potential outside the sphere will be the same as that from a point charge q' placed at the center of the sphere: $\Phi(r) = q'/r,\ \ r_0 \le r$. b) The potential of the sphere is then: $\Phi(r_0) = q'/r_0 = -q/R$

4 a) the contribution to the **E** field from the scalar potential is purely longitudinal. $-\nabla\Phi \to -i\mathbf{q}\Phi$. This is parallel to **q**. Thus it is longitudinal. Alternatively $\nabla \times (-\nabla\Phi) = 0$ since the curl of a gradient is always zero.This means that what is inside the parentheses is longitudinal.

b) the contribution to the **E** field from the vector potential is purely transverse.

$\nabla \cdot \left(-\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} \right) = -\frac{1}{c}\frac{\partial \left(\overbrace{\nabla \cdot \mathbf{A}}^{0} \right)}{\partial t} = 0$. Thus what is inside the parentheses is transverse.

c) the **B** field is purely transverse.
$\nabla \cdot \mathbf{B} = \nabla \cdot (\nabla \times \mathbf{A}) = 0$, since the divergence of a curl is always zero. Hence, the **B** fields is transverse.