1) There will be an induced surface charge density on the surface of the sphere that produces exactly the same potential outside the sphere as a mirror charge. Thus outside the sphere the potential is that from the charge q and the mirror charge. The mirror charge has the charge *-aq/b* and is placed at the distance a^2/b from the origin. Do we have any monopole contribution? Yes, there is a net charge and the monopole contribution is

$\Phi^{(1)}(r,\theta,\varphi)=\left[q-(aq/b)\right]/r=q\left[1-(a/b)\right]/r$. Do we have any dipole contribution? Yes the dipolemoment is: $\mathbf{p}=\hat{z}\left[qb-(aq/b)\left(a^2/b\right)\right]=\hat{z}qb\left[1-\left(a^3/b^3\right)\right]$ and $\Phi^{(2)}(r,\theta,\varphi)=\mathbf{p}\cdot\mathbf{r}/r^3=pr\cos\theta/r^3=qb\left[1-\left(a^3/b^3\right)\right]\cos\theta/r^2$

Do we have axial symmetry? Yes, and

$$Q=q\left(3b^2-b^2\right)-(aq/b)\left[3\left(a^2/b\right)^2-\left(a^2/b\right)^2\right]=q\left[2b^2-2a^5/b^3\right]=2qb^2\left[1-(a/b)^5\right];\Phi^{(4)}(r,\theta,\varphi)=Q\left(3\cos^2\theta-1\right)/4r^3$$

$$\Phi(r,\theta,\varphi)=\underline{q\left[1-(a/b)\right]/r+qb\left[1-\left(a^3/b^3\right)\right]\cos\theta/r^2+2qb^2\left[1-(a/b)^5\right]\left(3\cos^2\theta-1\right)/4r^3}$$

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2. The potential from a line charge is obtained from Gauss' law:

$2\pi r\Delta lD(r)=4\pi\sigma_l\Delta l\rightarrow E(r)=D(r)=2\sigma_l\Delta l/(r\Delta l)=\sigma_l/r\rightarrow\Phi(r)=-2\sigma_l\ln(r)=-\sigma_l\ln\left(r^2\right)$

We are supposed to show that the total potential outside the cylinder can be written as:

$$\Phi(\mathbf{r})=-\sigma_l\ln\left[x^2+(y-b)^2\right]+\sigma_l\ln\left[x^2+\left(y-a^2/b\right)^2\right]+C_1$$

$$=-\sigma_l\ln\left\{\left[x^2+(y-b)^2\right]\Big/\left[x^2+\left(y-a^2/b\right)^2\right]\right\}+C_1=-\sigma_l\ln\left\{C_2\left[x^2+(y-b)^2\right]\Big/\left[x^2+\left(y-a^2/b\right)^2\right]\right\}$$

We should show that the proper choice of constant leads to the potential zero on the whole cylinder surface.

$$\Phi(r=a)=\left|x^2=a^2-b^2\right|=-\sigma_l\ln\left\{C_2\left[a^2-y^2+(y-b)^2\right]\Big/\left[a^2-y^2+\left(y-a^2/b\right)^2\right]\right\}$$

$$=-\sigma_l\ln\left\{C_2\left[a^2+b^2-2yb\right]\Big/\left[a^2+a^4/b^2-2ya^2/b\right]\right\}$$

$$=-\sigma_l\ln\left\{C_2\left[a^2+b^2-2yb\right]\Big/\frac{a^2}{b^2}\left[b^2+a^2-2yb\right]\right\}=-\sigma_l\ln\left\{C_2\frac{b^2}{a^2}\right\}=0\rightarrow C_2=\frac{a^2}{b^2}\rightarrow C_1=\underline{\sigma_l\ln\left(\frac{b^2}{a^2}\right)}$$

3.

$$\Phi(u,v)=-V+\frac{2V}{\pi}v=-V+\frac{2V}{\pi}\arg(z)=-V+\frac{2V}{\pi}\tan^{-1}(y/x)\rightarrow\mathbf{E}=-\nabla\Phi=-\frac{2V}{\pi}\frac{1}{1+(y/x)^2}\left(-y/x^2,1/x\right)$$

$$\rightarrow E_y\left(x,0^+\right)=\frac{-2V}{\pi x}\rightarrow\sigma=\frac{-2V}{\pi x4\pi}=\underline{\frac{-V}{2\pi^2}\frac{1}{x}}$$

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4. The velocity of the charges (drift velocity due to the rotation) is $\mathbf{v}=\omega R\hat{\varphi}$. They are influenced by a Lorentz force $\mathbf{F}=q\mathbf{v}\times\mathbf{B}/c=q\omega R\hat{\varphi}\times B\hat{z}/c=q\omega RB\hat{\varphi}\times\hat{z}/c=q\omega RB\hat{R}/c$. An EMF is developed *from the center towards the rim*: $\varepsilon=\frac{1}{c}\int_0^a\omega RBdR=\omega Ba^2/2c$. In other words there will be a potential difference of this size across the resistor and the resulting current is: $\underline{I=\omega Ba^2/2Rc}$. If we do not have a resistor a large current can be generated. It is limited by the total resistance in the circuit. The power loss in the resistor is $RI^2=I\,\omega Ba^2/2c$ which is equal to the supplied power, i.e., the torque times the angular velocity: $N\omega=I\,\omega Ba^2/2c\rightarrow\underline{N=I\,Ba^2/2c}$

Here we can imagine some applications. If the **B**-field is known we may have an ampere-meter displaying torque. Alternatively one may measure magnetic induction if one knows *I* and *N*.