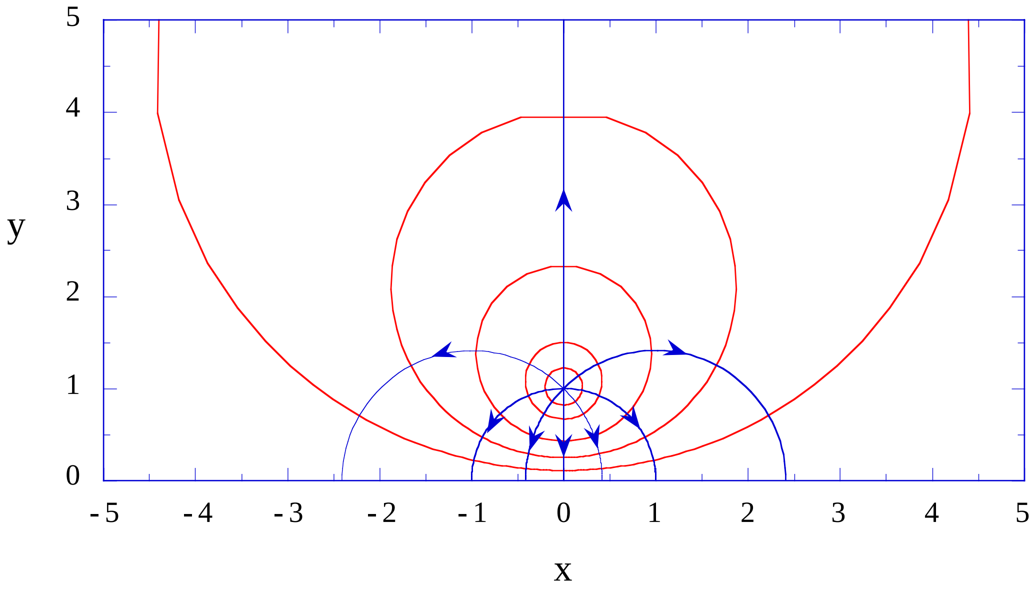
TFYY67

ELECTRODYNAMICS

ELEKTROMAGNETISK FÄLTTEORI OCH VÅGUTBREDNING

http://www.ifm.liu.se/courses/TFYY67/index.html



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Rules:

16 occasions
lectures mixed with problem solving sessions

Examination:

Written examination with 4 problems giving possible 4x4=16 points.

4 home work problems (HW) giving possible 4x1=4 points. Each home work problem will have a strict dead line.

These will be counted together with the result from the written examination. The credit for the homework problems is time limited. Next year a new set of homework problems have to be solved.

Allowed aid during the examination are:
The text book
Physics Handbook
Electronic calculator
English/Swedish dictionary

<u>Very important</u>: Since the text book is allowed during the examination you must not make any notes in the book! Remember this!

A short course of 6 chapters, KKKA, in complex analysis is posted on the web page for those who want to refresh their knowledge. It is written in Swedish.

Fundamental equations, **Maxwell's equations**

$$\nabla \cdot \mathbf{D} = 4\pi\rho \qquad \textit{Gauss' law}$$

$$\nabla \cdot \mathbf{B} = 0 \qquad \textit{Absence of free magnetic poles}$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \qquad \textit{Faraday's law}$$

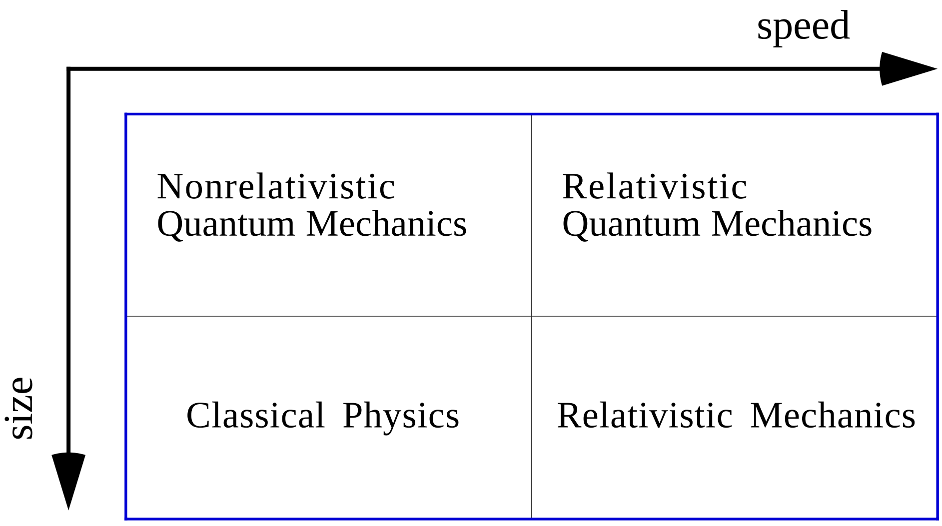
$$\nabla \times \mathbf{H} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t} \qquad \textit{Ampere's law}$$

We will derive these in the lectures that follow.

ρ		$\mathbf{J}$	
ρ_b		$\mathbf{J}_b$	
ρ_{cond}	ρ_{free}	$\mathbf{J}_{cond}$	$\mathbf{J}_{free}$
$\rho_{external}$		$\mathbf{J}_{external}$	

These equations have a very rich spectrum of solutions. In this course we will study four aspects:

- **Generation of electromagnetic waves**
- **Propagation of these waves in space;**
  in empty space; inside materials
- **Their interaction with matter of various forms**
- **How matter is affected by these waves:** Surface energy, surface tension, van der Waals force, Casimir force.



What is surprising is that Maxwell's equations look the same in all four corners.

E Electric field (statvolt/cm)
D Displacement (statvolt/cm)
B Magnetic induction (gauss)
H Magnetic field (oersted)

E and **B** are the fundamental fields and **D** and **H** are auxiliary fields (help fields)

Gaussian units   CGS-units

Benefits as compared to **SI-units** or rationalized **MKSA-units**:

Factors of speed of light appear explicitly and appropriately
E and **D**, as well as **H** and **B**, become identical in vacuum.
There are no "strange" ε_0 or μ_0 appearing.
E and **B** have the same amplitudes for plane waves in vacuum.
Easy to convert into atomic units: just let e, m and $\hbar$ be unity.
Relativistic Electrodynamics is easily formulated.

Drawbacks:

SI is the system of legal metrology.
SI-system preferred by most people not doing actual calculations within condensed matter.

The units for different quantities in the two systems are given in Appendix D

Electromagnetic equations in the two systems are compared in Appendix E.

How are unit systems constructed?

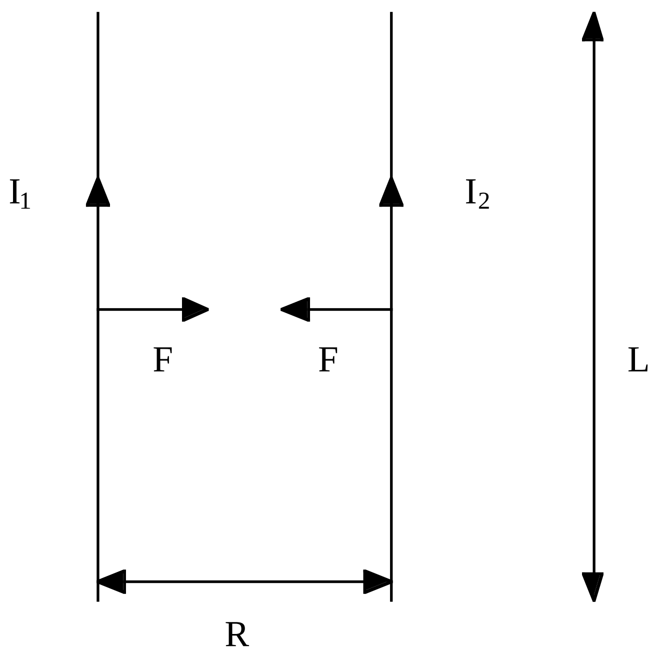
One starts from two experiments

1) The force between two charges

$$F = k_e \frac{Q_1 Q_2}{r^2}$$

2) The force between two long, parallel, current carrying wires.

$$F = k_m \frac{2 I_1 I_2}{R} L$$



k_e and k_m are two constants, where $k_e / k_m = c^2$; c = speed of light in vacuum. One of them may be chosen at will.

In the construction of the SI-system one chose the value of k_m by determining the unit of current, Ampère, by stating that the force per meter wire, for wires separated by 1 m is $2 \times 10^{-7} N$ when the current of 1 A is running through both wires. This seems like an odd choice to make!

With this choice the currents running in our everyday wires are of the order of 1 A. This is the only reason to make this choice. This leads to

$$k_m = 10^{-7}\, Vs/Am \Rightarrow k_e = 10^{-7} c^2\, Vs/Am$$

These are awkward constants to carry around in the equations. Instead one introduced

$$\begin{cases} k_e = 1/4\pi\varepsilon_0\,; \\ k_m = \mu_0/4\pi \end{cases} \rightarrow \mu_0\varepsilon_0 = 1/c^2$$

$\varepsilon_0 = 8.8541878 \times 10^{-12}\, As/Vm$, dielectric constant for vacuum
$\mu_0 = 4\pi \times 10^{-7}\, Vs/Am$, magnetic permeability of vacuum.

These numbers have nothing to do with some properties of vacuum. They are simply the result of the definition of the SI-unit system.

In CGS one chooses $k_e = 1$ and $k_m = 1/c^2$ and no mysterious ε_0 and μ_0 appear.

The basic units in SI are m, kg, s and A(mpère).

The basic units in CGS are cm, g, s and statcoulomb.

Quantity	CGS	SI
Velocity of Light	c	$\sqrt{1/\mu_0\varepsilon_0}$
Electric Field (potential, voltage)	$\mathbf{E}(\Phi, V)$	$\sqrt{4\pi\varepsilon_0}\,\mathbf{E}(\Phi, V)$
Displacement	$\mathbf{D}$	$\sqrt{4\pi/\varepsilon_0}\,\mathbf{D}$
Charge Density (charge, current density, current, polarization)	$\rho(q, \mathbf{J}, \mathbf{I}, \mathbf{P})$	$\sqrt{1/4\pi\varepsilon_0}\,\rho(q, \mathbf{J}, \mathbf{I}, \mathbf{P})$
Magnetic Induction	$\mathbf{B}$	$\sqrt{4\pi/\mu_0}\,\mathbf{B}$
Magnetic Field	$\mathbf{H}$	$\sqrt{4\pi\mu_0}\,\mathbf{H}$
Magnetization	$\mathbf{M}$	$\sqrt{\mu_0/4\pi}\,\mathbf{M}$
Conductivity (capacitance)	$\sigma(C)$	$\sigma(C)/4\pi\varepsilon_0$
Dielectric Function	ε	$\varepsilon/\varepsilon_0 = \varepsilon_r$
Permeability	μ	$\mu/\mu_0 = \mu_r$
Resistance (impedance, inductance)	$R(Z, L)$	$4\pi\varepsilon_0 R(Z, L)$

The vector calculus we need is summarized in Appendix A of the book. The most important relations are given on the inside of the cover of the book.

Notation:

gradient:	**grad** or ∇	(gradienten)
divergence:	div or $\nabla\cdot$	(divergensen)
curl:	**curl** or $\nabla\times$	(rotationen)

Gauss' theorem: $\int_V \nabla\cdot\mathbf{A}dv = \oint_S \mathbf{A}\cdot\mathbf{n}dS$

Stokes' theorem: $\int_S \nabla\times\mathbf{A}\cdot\mathbf{n}dS = \oint_\Gamma \mathbf{A}\cdot d\mathbf{l}$

The Fourier transform can be defined in different ways. We define the Fourier transforms with respect to position and time and their inverses in the following way:

$$f(\mathbf{q}) = \int d^3 r e^{-i\mathbf{q}\cdot\mathbf{r}} f(\mathbf{r})$$

$$f(\mathbf{r}) = \frac{1}{\Omega}\sum_{\mathbf{q}} e^{i\mathbf{q}\cdot\mathbf{r}} f(\mathbf{q}) = \int \frac{d^3 q}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{r}} f(\mathbf{q})$$

$$f(\omega) = \int_{-\infty}^{\infty} dt e^{i\omega t} f(t)$$

$$f(t) = \int_{-\infty}^{\infty} \frac{d\omega}{(2\pi)} e^{-i\omega t} f(\omega)$$

$$f(\mathbf{q},\omega) = \int d^3 r \int_{-\infty}^{\infty} dt e^{-i(\mathbf{q}\cdot\mathbf{r}-\omega t)} f(\mathbf{r},t)$$

$$f(\mathbf{r},t) = \int \frac{d^3 q}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d\omega}{(2\pi)} e^{i(\mathbf{q}\cdot\mathbf{r}-\omega t)} f(\mathbf{q},\omega)$$

With these sign conventions, Fourier transforming differential equations has the following substitutional effects:

$$\frac{\partial}{\partial t} \rightarrow -i\omega$$

$$\nabla\cdot \rightarrow i\mathbf{q}\cdot$$

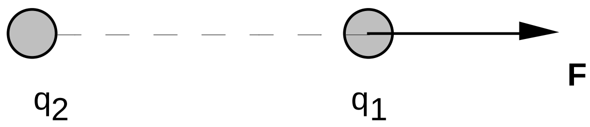
$$\nabla\times \rightarrow i\mathbf{q}\times$$

Ch. 1 Fundamentals of Static Electromagnetism

Coulomb's force law:

The force between two point charges at rest. The force exerted on q_1 by q_2:

$$\boxed{\mathbf{F}_{12} = \frac{q_1 q_2}{r^2} e_{\mathbf{r}}}$$



Empirical law found by Coulomb in 1785. He found the inverse power of *r* to be 2 ± 0.02. New measurements have improved the accuracy:

Experimental test of Coulomb's law: $E = q/r^{2+\delta}$

Experimenter (date)	Apparatus or geometry	Upper limit for the parameter δ
Robison (1769)	Gravitational torque on a pivot arm	0.06
Cavendish (1773)	Two concentric metal spheres	0.02
Coulomb (1785)	Torsion balance	0.04
Maxwell (1873)	Two concentric spheres	$\frac{1}{21\,600}$
Plimpton and Lawton (1938)	Two concentric spheres	2×10^{-9}
Cochran (1967)	Concentric cubical conductors	9.2×10^{-12}
Bartlett, Goldhagen, and Phillips (1970)	Five concentric spheres	1.3×10^{-13}
Williams, Faller, and Hill (1971)	Five concentric icosahedrons	$(2.7\pm3.1)\times10^{-16}$
Fulcher (1986)	Reevaluation of the experiment above.	$(1.0\pm1.2)\times10^{-16}$

From: Lewis P. Fulcher, Physical Review **A33**, 759 (1986).

Note: The forces between two point charges in motion are not always pointing along the line connecting the two charges. The law of action and reaction is still obeyed. The two forces are equal in magnitude and pointing in opposite directions.

Since the force on a point charge in an electric field is

$$\mathbf{F} = q\mathbf{E}, \quad \text{(definition of } \mathbf{E}\text{)}$$

the electric field from charge 2 at the position of charge 1 is

$$\mathbf{E}_{12} = \frac{q_2}{r^2} e_{\mathbf{r}}$$

or more general,

Coulomb's field law:

$$\boxed{\mathbf{E} = \frac{q'}{r^2} e_{\mathbf{r}}}$$

The electric field in **r** due to a source charge q'. The vector **r** is the vector from the source point to the field point and it is pointing along the unit vector $e_{\mathbf{r}}$.

The above relation is linear which means that **the principle of superposition** applies. If we go ahead and study Maxwell's equations we find that they are all linear.

The general form of Coulomb's law for an arbitrary array of point charges is

$$\boxed{\mathbf{E}(\mathbf{r}) = \sum_i \frac{q(\mathbf{r}'_i)}{|\mathbf{r} - \mathbf{r}'_i|^2} e_{\mathbf{r}-\mathbf{r}'_i}}$$

or for a continuous distribution of charges:

$$\boxed{\mathbf{E}(\mathbf{r}) = \int d^3r' \frac{\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^2} e_{\mathbf{r}-\mathbf{r}'} = \int d^3r' \frac{\rho(\mathbf{r}')(\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3}}$$

Here we should note that the field at position **r** depends on the charge distribution in all points in space. The above integral is a so-called *convolution integral*.

$$F(\mathbf{r}) = \int d^3\mathbf{r}' g(\mathbf{r} - \mathbf{r}')h(\mathbf{r}') \rightarrow F(\mathbf{q}) = g(\mathbf{q})h(\mathbf{q})$$

We will return to convolution integrals later and their properties.

One can show (Problem 1-3 in the text book) that the total flux of the electric field through a closed surface (Gaussian surface) is 4π times the total enclosed charge, i.e.,

$$\oint_S \mathbf{E} \cdot d\mathbf{a} = 4\pi q_{encl}$$

This is Gauss' law on integral form.

Using Gauss' theorem:

$$\int_V div\mathbf{E}dv = \oint_S \mathbf{E} \cdot \mathbf{n}dS = \underset{\leftarrow}{} \left[\oint_S \mathbf{E} \cdot d\mathbf{a} = 4\pi q_{encl}\right]_{\rightarrow} = \int_V 4\pi\rho dv$$

gives

$$\int_V div\mathbf{E}dv = \int_V 4\pi\rho dv$$

This is valid for all volumes. This means that the integrands have to be identical. Thus

$\boxed{div\mathbf{E} = 4\pi\rho}$ Gauss' law on differential form 1st ME

Note here that this relation is local. Both $\mathbf{E}$ and ρ are given in the same space point. This might seem a little bit strange since we showed above that the electric field depends on the charge distribution over all space.
This is the first of Maxwell's equations. It is valid outside a medium. It is also valid inside a medium if the charge density also includes the bound polarization charges of the medium.

The Coulomb field is a central force from which follows that the field is conservative, i.e.,

$\oint \mathbf{E} \cdot d\mathbf{l} = 0$ Conservative law on integral form

Stokes' theorem gives:

$$0 = \oint \mathbf{E} \cdot d\mathbf{l} = \int_S \mathbf{curl}\,\mathbf{E} \cdot \mathbf{n}\,dS$$

is valid for all contours. This means that

$\boxed{\mathbf{curl}\ \ \mathbf{E} = 0}$ Conservative law on differential form 3rd ME

This is what the third of Maxwell's equations is reduced to for static charge distributions and stationary current distributions. (In the case of time dependent currents the electric field is no longer conservative.)

Since the field is conservative we may define a scalar potential Φ for the electric field:

$$\Phi(\mathbf{r}) = -\int_{\mathbf{r}_0}^{\mathbf{r}} \mathbf{E} \cdot d\mathbf{l}$$

or

$$\boxed{\mathbf{E} = -\mathbf{grad}\,\Phi}$$

Inserting this relation into Gauss' equation gives

$$\mathrm{div}\,\mathbf{grad}\,\Phi = -4\pi\rho$$

or

$\boxed{\nabla^2\Phi = -4\pi\rho}$ Poisson's equation

Note that also here the source and field points are the same.

If there are no charges at the field point the field obeys the equation:

$$\boxed{\nabla^2 \Phi = 0} \quad \text{Laplace's equation}$$

The potential from a point charge can be obtained from

$$-\mathbf{grad}\,\Phi = \mathbf{E} = \frac{q}{r^2}\mathbf{e_r}$$

and hence

$$\Phi = \frac{q}{r}$$

Superposition gives

$$\Phi(\mathbf{r}) = \int d^3r' \frac{\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}$$

The potential depends on the charge density in the whole space.

Suppose we wish to calculate the *potential* at a point *due to* a given distribution of *source charges*. This can be done in several different ways. Some of these are:

1. Superpose the **E** fields from the source charges,

$$\mathbf{E}(\mathbf{r}) = \int d^3r' \frac{\rho(\mathbf{r}')(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3},$$

and then perform the integral $\Phi(\mathbf{r}) = -\int_{\mathbf{r}_0}^{\mathbf{r}} \mathbf{E}\cdot d\mathbf{l}$

2. Superpose the potential from all source charges,

$$\Phi(\mathbf{r}) = \int d^3r' \frac{\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}$$

3. Solve Poisson's equation,

$$\nabla^2\Phi = -4\pi\rho$$

We will do this extensively in chapter 3.

4. Calculate the Fourier transform of the charge distribution, $\rho(\mathbf{q})$ and then find the potential as the inverse Fourier transform of $4\pi\rho(\mathbf{q})/q^2$.

5. Add a test charge q in $\mathbf{r}$ and determine the total electric field from the source charges and the test charge. Write down the expression for the total energy stored in the electric field as an integral of the energy density over the whole space. Subtract the contribution from all individual charges (the self-interaction energies). This gives the interaction energy. The potential in $\mathbf{r}$ is then:

$$\Phi(\mathbf{r}) = \frac{d}{dq}U$$

The result is

$$\Phi(\mathbf{r}) = \frac{1}{8\pi}2\int\int d^3r'd^3r'' \frac{\mathbf{r}'-\mathbf{r}}{|\mathbf{r}'-\mathbf{r}|^3}\frac{\mathbf{r}'-\mathbf{r}''}{|\mathbf{r}'-\mathbf{r}''|^3}\rho(\mathbf{r}'')$$

If we want to find the *electric field from the source charges* we can do that in three different ways:

(1) Directly using

$$\mathbf{E}(\mathbf{r}) = \int d^3r' \frac{\rho(\mathbf{r}')(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}$$

(2) Calculating the scalar potential using one of the many suggestions above, and then using the equation:

$$\mathbf{E} = -\mathbf{grad}\,\Phi$$

(3) Using the two differential equations:

$$\mathbf{curl}\;\;\mathbf{E} = 0;$$

$$div\mathbf{E} = 4\pi\rho$$