

**Formelblad - Elektromagnetism D.**

Maxwells ekvationer

$$\begin{array}{ll}
 1 \quad \oint_S \mathbf{D} \cdot d\mathbf{S} = Q & 2 \quad \oint_C \mathbf{E} \cdot d\mathbf{l} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S} \\
 3 \quad \oint_S \mathbf{B} \cdot d\mathbf{S} = 0 & 4 \quad \oint_C \mathbf{H} \cdot d\mathbf{l} = \int_S \mathbf{J} \cdot d\mathbf{S} + \int_S \frac{\partial \mathbf{D}}{\partial t} \cdot d\mathbf{S}
 \end{array}$$

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$$

$$\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M})$$

Potential och  $\mathbf{E}$ -fält från elektriskt dipolmoment

$\mathbf{B}$ -fält från magnetiskt dipolmoment

$$V = \frac{p \cdot \cos \theta}{4\pi\epsilon_0 \cdot r^2}$$

$$\mathbf{B} = \frac{\mu_0 m}{4\pi \cdot r^3} (2 \cos \theta \cdot \hat{\mathbf{r}} + \sin \theta \cdot \hat{\boldsymbol{\theta}})$$

$$\mathbf{E} = \frac{p}{4\pi\epsilon_0 \cdot r^3} (2 \cos \theta \cdot \hat{\mathbf{r}} + \sin \theta \cdot \hat{\boldsymbol{\theta}})$$

**Biot-Savarts lag**

$$\mathbf{B} = \oint_C \frac{\mu_0 I d\mathbf{l} \times \hat{\mathbf{r}}}{4\pi r^2}; \quad \mathbf{B} = \oint_{\tau} \frac{\mu_0 \mathbf{J} \times \hat{\mathbf{r}} d\tau}{4\pi r^2}; \quad \mathbf{B} = \oint_S \frac{\mu_0 \mathbf{J}_s \times \hat{\mathbf{r}} dS}{4\pi r^2}$$

Gradient i olika koordinatsystem

$$grad(V) = \frac{\partial V}{\partial x} \cdot \hat{\mathbf{x}} + \frac{\partial V}{\partial y} \cdot \hat{\mathbf{y}} + \frac{\partial V}{\partial z} \cdot \hat{\mathbf{z}} \quad \text{cartesiskt}$$

$$grad(V) = \frac{\partial V}{\partial R} \cdot \hat{\mathbf{R}} + \frac{1}{R} \cdot \frac{\partial V}{\partial \phi} \cdot \hat{\boldsymbol{\phi}} + \frac{\partial V}{\partial z} \cdot \hat{\mathbf{z}} \quad \text{cylindriskt}$$

$$grad(V) = \frac{\partial V}{\partial r} \cdot \hat{\mathbf{r}} + \frac{1}{r} \cdot \frac{\partial V}{\partial \theta} \cdot \hat{\boldsymbol{\theta}} + \frac{1}{r \sin \theta} \cdot \frac{\partial V}{\partial \phi} \cdot \hat{\boldsymbol{\phi}} \quad \text{sfäriskt}$$

Omvandling av rörliga koordinater till cartesiska.

$$\begin{array}{ll}
 \hat{R} = \cos \phi \hat{x} + \sin \phi \hat{y} & \hat{r} = \sin \theta \cos \phi \hat{x} + \sin \theta \sin \phi \hat{y} + \cos \theta \hat{z} \\
 \hat{\phi} = -\sin \phi \hat{x} + \cos \phi \hat{y} & \hat{\boldsymbol{\theta}} = \cos \theta \cos \phi \hat{x} + \cos \theta \sin \phi \hat{y} - \sin \theta \hat{z} \\
 \hat{z} = \hat{z} & \hat{\boldsymbol{\phi}} = -\sin \phi \hat{x} + \cos \phi \hat{y}
 \end{array}$$

Några vanliga integraler

$$\int \frac{dx}{(x^2 + a^2)^{1/2}} = \ln \left[ x + (x^2 + a^2)^{1/2} \right]; \quad \int \frac{dx}{(x^2 + a^2)} = \frac{1}{a} \arctan \left( \frac{x}{a} \right)$$

$$\int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{x}{a^2 (x^2 + a^2)^{1/2}}; \quad \int \frac{dx x^2}{(x^2 + a^2)} = x - a \arctan \left( \frac{x}{a} \right)$$

$$\int \frac{dx x^2}{(x^2 + a^2)^{3/2}} = \frac{-x}{(x^2 + a^2)^{1/2}} + \ln \left[ x + (x^2 + a^2)^{1/2} \right]$$

Energitäthet:  $\frac{\partial W_{em}}{\partial \tau} = \frac{1}{2} \mathbf{E} \cdot \mathbf{D} + \frac{1}{2} \mathbf{B} \cdot \mathbf{H}.$

Ljushastighet i dielektriskt medium:  $c_m = (\epsilon_0 \epsilon_r \mu_0)^{-1/2} = c_0 / \sqrt{\epsilon_r}.$

Brytningsindex:  $n = c_0 / c_m = \sqrt{\epsilon_r}.$

Intensitet hos plan elektromagnetisk våg:  $\mathbf{I} = \epsilon_0 c_0 n E_0^2 \sin^2(kx - \omega t) \cdot \hat{\mathbf{x}}.$

Telegrafekvationen för förlustfri ledning:

$$\frac{\partial^2 U}{\partial z^2} = L_l C_l \frac{\partial^2 U}{\partial t^2} \quad \text{alternativt} \quad \frac{\partial^2 i}{\partial z^2} = L_l C_l \frac{\partial^2 i}{\partial t^2}$$